On the Stability of a Planetary System Embedded in the β Pictoris Debris Disk

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Abstract. It has recently been stated that the warp structure observed around the star β Pictoris may be due to four planets embedded in its debris disk [1]. It, therefore, becomes important to investigate for what range of parameters, and for how long such a multibody system will be dynamically stable. We present the results of the numerical integration of the suggested planetary system for different values of the mass and radii of the planets, and their relative positions and velocities. We also present a statistical analysis of the results in an effort to understand the relation between the different regions of the parameter-space of the system and the duration of the orbital stability of the embedded planets.

INTRODUCTION

Beta Pictoris, a type A5 IV star at a distance of $\sim$19 pc from the Earth, is one of the youngest close stars with an approximate age of 12^{+8}_{-4} Myr [2]. Observational evidence indicates that this star is surrounded by a planetary debris disk [3, 4, 5]. The close proximity of such a young circumstellar disk has made β Pictoris an ideal candidate for the study of the evolution of protoplanetary disks and planetary system formation.

In the past several years, there have been a number of reports of the detection of symmetric warps in the β Pictoris debris disk [1, 5, 6, 7, 8]. Models have best explained these warps as gravitational perturbations caused by planetary companions [6, 7, 8]. Among these reports, the disk warps discovered by Wahhaj et al. [1] were explained as the edge-on projection of debris rings orbiting the central star. Models of the flux density allow some parameters of these rings to be determined via χ^2 fitting. Wahhaj et al. [1] proposed a multiple planet system to account for ring formation, noting that all adjacent rings are in mean-motion resonances.

In consideration of these findings, we undertake here a study of the dynamical evolution of a multibody system similar to that proposed by Wahhaj et al. [1]. We perform a statistical analysis of the stability of randomly generated systems within a portion of the total available parameter-space and briefly analyze the relationship between the parameters of the system and its orbital stability.

NUMERICAL ANALYSIS

The planetary system proposed by Wahhaj et al. [1] consists of four planets with radial distances and orbital inclinations equal to those of their corresponding warps [see 1, Table 1]. To study the dynamics of this planetary system, we explore a parameter-space which includes the mass number of the four planets, their radii[1], and their positions and velocities. The mass and radius of β Pictoris are taken to be $2.0\,M_\odot$ and $1.9\,R_\odot$, respectively [9].

We consider planets with masses randomly chosen between one to three Jupiter-masses. For each value of the mass of a planet, we calculate its radius assuming an average density equal to that of Jupiter (1.33 gcm^{-3}). We also assume that all planets are initially on direct Keplerian circular orbits and their orbital phases are chosen randomly from the range $0^\circ \le \phi \le 360^\circ$.

To explore the orbital stability of this planetary system, we integrated the system for 50 Myr using Mercury Integrator Package VI [10]. We considered the system to be stable if no planet came closer than three Hill's radii or obtained a radial distance larger than 1000 AU.

We ran a total of 20457 simulations using randomly generated values of planet masses and initial orbital phases. Of this total, 14409 simulations used unique parameter sets. The remaining 6048 systems were exact replications of the systems in the unique set—a consequence of the random number generation routine. The duplicate systems have been kept for the sake of statistical analysis, since they are randomly distributed throughout the phase space.

Table 1 shows the statistical data for all simulations grouped in five 10 Myr intervals. The middle two columns show the number and percentage of simulations that became unstable within their corresponding time intervals. The majority of the randomly generated systems became unstable at early stages of the integration, with their number decreasing as time increases. The rightmost column shows the percentage of the systems remaining stable beyond their respective time interval. As shown here, approximately 41% of the systems remained stable after the first 10 Myrs. From these systems, 6.7% were still stable after 20 Myr from the beginning of the integrations–the upper estimate of the lifetime of β Pictoris (20 Myr). During the last 10 Myr, only 0.1% of all systems were still stbale.

Figure 1 shows how stability lifetimes are distributed across a random sampling of the parameter-space. This histogram shows the number of systems that became unstable in 10^5 year intervals. It can be seen from this figure that for a system chosen randomly within our assumed parameter-space, it is most probable that the stability lifetime of the system is approximately 7 Myr. It is also seen that no system became unstable at an age of less than 1.0 Myr. We note that 15 systems remained stable for the entire 50 Myr and are therefore not accounted for in the histogram.

In a preliminary attempt to determine the relationship between initial conditions and stability lifetime, we have plotted the mass of each planet versus its initial phase-angle for those systems that remained stable for 50 Myr (Fig. 2). The initial phase of planet

[1] Planets' radii were needed to calculate their Hill's radii.

TABLE 1. Statistical data on five-body system stability.

Time (Myr)	Number of Unstable Systems	Percentage of Unstable Systems	Percentage of Remaining Stable Systems
0–10	12100	59.1	40.9
10–20	6980	34.1	6.7
20–30	1152	5.6	1.1
30–40	194	0.9	0.2
40–50	16	0.1	0.1

1 was set to 0° for all simulations. It is interesting to note that none of these systems contain a planet with a mass greater than $2.4 M_J$.

CONCLUSIONS

We have analyzed the stability of the proposed five-body planetary system embedded in the β Pictoris debris disk [1] for over 14000 initial conditions. Our results indicate that the majority of systems became unstable between a time of 1 to 10 million years. There were only 8 unique simulations that remained stable for the entire 50 Myr integration time. These systems contained planets with masses less than 2.4 times the mass of Jupiter.

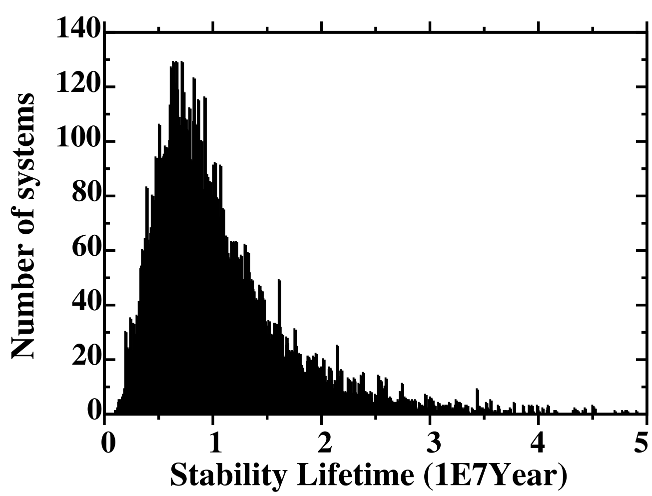

FIGURE 1. Histogram showing the number of systems that became unstable in 10^5 yr intervals. A randomly chosen system is most likely to become unstable near 7 Myr. No system in this sample became unstable in less than 1 Myr. The 15 systems that remained stable for the entire 50 Myr integration are not shown here.

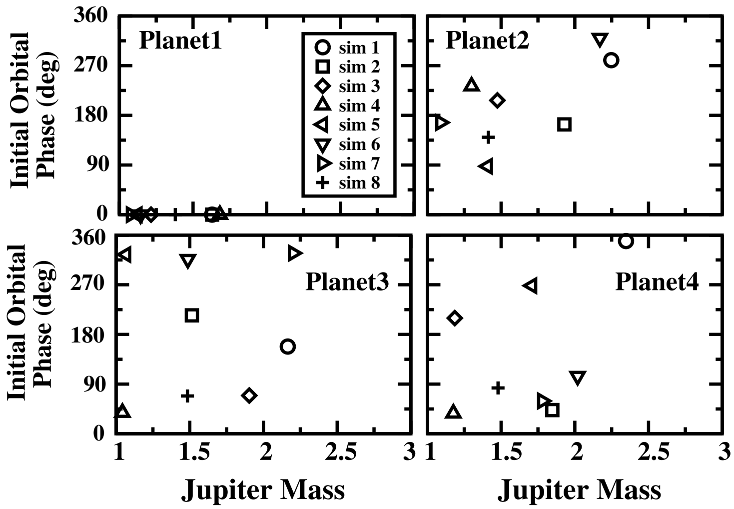

FIGURE 2. Mass versus initial phase for planets in the 8 unique systems that remained stable for the entire 50 Myr integration. Note that none of these planets have masses greater than 2.4 Jupite-mass.

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